

# THE OPTIMIST CLASSES

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# CSIR-NET-JRF RESULTS 2022



**ANNU**  
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**SAHIL RANA**  
**HR09000108**



**JAYESTHI**  
**RJ11000161**



**DASRATH**  
**RJ06000682**



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**SURYA PRATAP SINGH**  
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**UP10000095**



**CHANDAN**  
**RJ09000159**



**SAIKHOM JOHNSON**  
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**AJAY SAINI**  
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# THE OPTIMIST CLASSES

AN INSTITUTE FOR NET-JRF/GATE/IIT-JAM/JEST/TIFR/M.Sc ENTRANCE EXAMS

CONTACT: 9871044043

## IIT-JAM 2021 PAPER

1. The function  $e^{\cos x}$  is Taylor expanded about  $x = 0$ . The coefficient of  $x^2$  is

(a)  $-\frac{1}{2}$  (b)  $-\frac{e}{2}$   
(c)  $\frac{e}{2}$  (d) Zero

2. Let  $M$  be  $2 \times 2$  matrix. Its trace is 6 and its determinant has value 8. Its eigenvalues are

(a) 2 and 4 (b) 3 and 3  
(c) 2 and 6 (d) -2 and -3

3. A planet is in a highly eccentric orbit about a star. The distance of its closest approach is 300 times smaller than its farthest distance from the star. If

the corresponding speed are  $v_c$  and  $v_f$ , then  $\frac{v_c}{v_f}$

is

(a)  $\frac{1}{300}$  (b)  $\frac{1}{\sqrt{300}}$   
(c)  $\sqrt{300}$  (d) 300

4. An object of density  $\rho$  is floating in a liquid with 75% of its volume submerged. The density of the liquid is

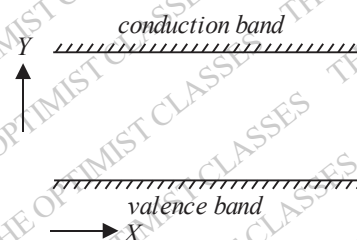
(a)  $\frac{4}{3}\rho$  (b)  $\frac{3}{2}\rho$   
(c)  $\frac{8}{5}\rho$  (d)  $2\rho$

5. An experiment with a Michelson interferometer is performed in vacuum using a laser of wavelength  $610 \text{ nm}$ . One of the beams of the interferometer

passes through a small glass cavity  $1.3 \text{ cm}$  long. After the cavity is completely filled with a medium of refractive index  $n$ , 472 dark fringes are counted to move past a reference line. Given that the speed of light is  $3 \times 10^8 \text{ m/s}$ , the value of  $n$  is

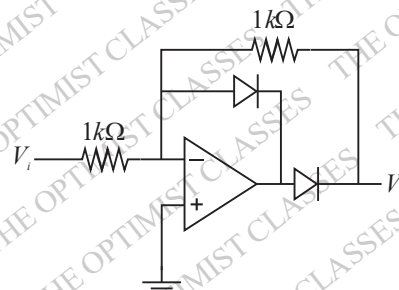
(a) 1.01 (b) 1.04  
(c) 1.06 (d) 1.10

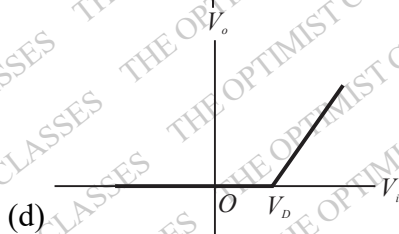
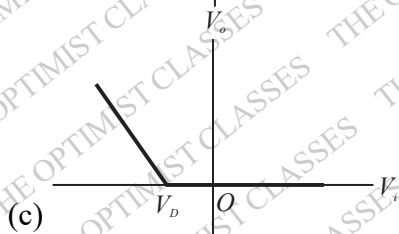
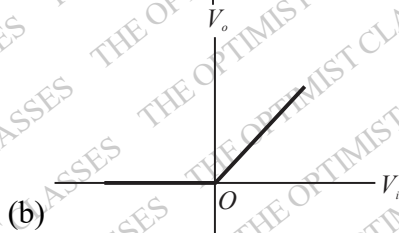
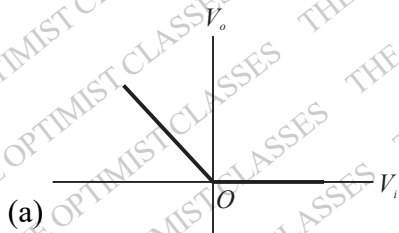
6. For a semiconductor material, the conventional flat band energy diagram is shown in the figure. The variables  $Y, X$ , respectively, are



(a) Energy, Momentum  
(b) Energy, Distance  
(c) Distance, Energy  
(d) Momentum, Energy

7. For the given circuit,  $V_D$  is the threshold voltage of the diode. The graph that best depicts the variation of  $V_o$  with  $V_i$  is





8. Arrange the following telescopes, where  $D$  is the telescope diameter and  $\lambda$  is the wavelength, in order of decreasing resolving power:

(I)  $D = 100m, \lambda = 21cm$

(II)  $D = 2m, \lambda = 500nm$

(III)  $D = 1m, \lambda = 100nm$

(IV)  $D = 2m, \lambda = 10mm$

(a) III, II, IV, I (b) II, III, I, IV

(c) IV, III, II, I (d) III, II, I, IV

9. Metallic lithium has *bcc* crystal structure. Each unit cell is a cube of side  $a$ . The number of atoms per unit volume is

(a)  $\frac{1}{a^3}$

(b)  $\frac{2}{\sqrt{2}a^3}$

(c)  $\frac{2}{a^3}$

(d)  $\frac{4}{a^3}$

10. The moment of inertia of a solid sphere (radius  $R$  and mass  $M$ ) about the axis which is at a distance of  $\frac{R}{2}$  from the center is

(a)  $\frac{3}{20}MR^2$

(b)  $\frac{1}{2}MR^2$

(c)  $\frac{13}{20}MR^2$

(d)  $\frac{9}{10}MR^2$

11. Let  $(x, y)$  denote the coordinates in a rectangular Cartesian coordinate system  $C$ . Let  $(x', y')$  denote the coordinates in another coordinate system  $C'$ , defined by

$x' = 2x + 3y$

$y' = -3x + 4y$

The area element in  $C'$ , is

(a)  $\frac{1}{17}dx'dy'$

(b)  $12dx'dy'$

(c)  $dx'dy'$

(d)  $x'dx'dy'$

12. Three events,  $E_1 (ct = 0, x = 0)$ ,  $E_2 (ct = 0, x = L)$  and  $E_3 (ct = 0, x = -L)$  occur, as observed in an inertial frame  $S$ . Frame  $S'$  is moving with a speed  $v$  along the positive  $x$ -direction with respect to  $S$ . In  $S'$ , let  $t'_1, t'_2, t'_3$  be the respective times at which  $E_1, E_2$ , and  $E_3$  occurred. Then,

(a)  $t'_2 < t'_1 < t'_3$

(b)  $t'_1 = t'_2 = t'_3$

(c)  $t'_3 < t'_1 < t'_2$

(d)  $t'_3 < t'_2 < t'_1$

13. The solution  $y(x)$  of the differential equation

$y \frac{dy}{dx} + 3x = 0, y(1) = 0$ , is described by

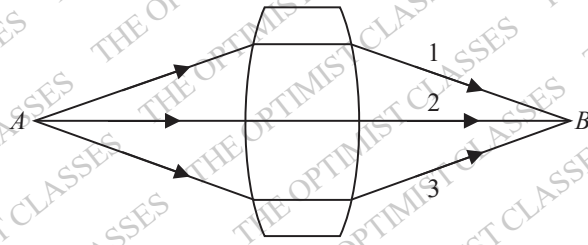
(a) an ellipse

(b) a circle

(c) a parabola

(d) a straight line

14. In the figure below, point  $A$  is the object and point  $B$  is the image formed by the lens. Let  $l_1, l_2$  and  $l_3$  denote the optical path lengths of the three rays 1, 2 and 3, respectively. Identify the correct statement,



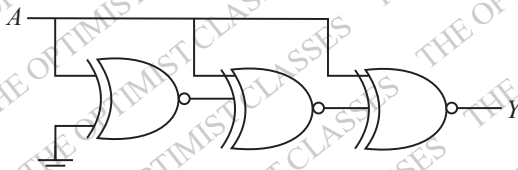
- (a)  $l_1 = l_2 = l_3$  (b)  $l_1 > l_2 < l_3$   
(c)  $l_1 = l_3 < l_2$  (d)  $l_1 = l_3 > l_2$

15. A particle, initially at the origin in an inertial frame  $S$ , has a constant velocity  $V\hat{i}$ . Frame  $S'$  is rotating about the  $z$ -axis with angular velocity  $\omega$  (anticlockwise). The coordinate axes of  $S'$  coincide with those of  $S$  at  $t = 0$ . The velocity of

the particle  $(V'_x, V'_y)$  in the  $S'$  frame, at  $t = \frac{\pi}{2\omega}$  is

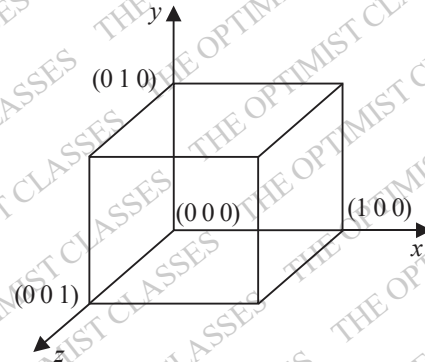
- (a)  $\left(-\frac{V\pi}{2}, -V\right)$  (b)  $(-V, -V)$   
(c)  $\left(\frac{V\pi}{2}, -V\right)$  (d)  $\left(\frac{3V\pi}{2}, -V\right)$

16. For the given circuit, the output  $Y$  is



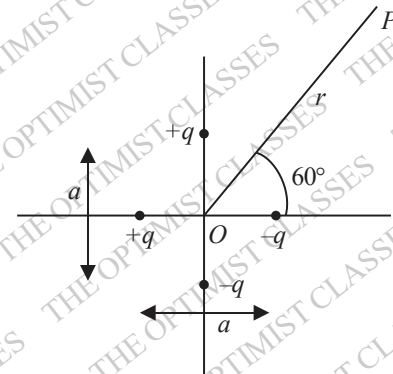
- (a) 0 (b) 1  
(c) A (d)  $\bar{A}$

17. The total charge contained within the cube (see figure), in which the electric field is given by  $\vec{E} = K(4x^2\hat{i} + 3y\hat{j})$ , where  $\epsilon_0$  is the permittivity of free space, is



- (a)  $7K\epsilon_0$  (b)  $5K\epsilon_0$   
(c)  $3K\epsilon_0$  (d) Zero

18. Four charges are placed very close to each other, as shown. The separation between the two charges on the  $y$ -axis is  $a$ . The separation between the two charges on the  $x$ -axis is also  $a$ . The leading order (non-vanishing) form of the electrostatic potential, at point  $P$ , at a distance  $r$  from the origin ( $r \gg a$ ), is



- (a)  $\frac{1}{4\pi\epsilon_0} \frac{qa}{2r^2} (\sqrt{3} - 1)$   
(b)  $\frac{1}{4\pi\epsilon_0} \frac{2qa}{r^2}$   
(c)  $\frac{1}{4\pi\epsilon_0} \frac{qa}{r^2} (\sqrt{5} - 1)$   
(d)  $\frac{1}{4\pi\epsilon_0} \frac{qa}{r^2} (1 - \sqrt{3})$

19. At  $t = 0$ ,  $N_0$  number of a radioactive nuclei  $A$



start decaying into  $B$  with a decay constant  $\lambda_a$ .

The daughter nuclei  $B$  decay into nuclei  $C$  with a decay constant  $\lambda_b$ . Then, the number of nuclei  $B$  at small time  $t$  (to the leading order) is

- (a)  $\lambda_a N_0 t$  (b)  $(\lambda_a - \lambda_b) N_0 t$   
 (c)  $(\lambda_a + \lambda_b) N_0 t$  (d)  $\lambda_b N_0 t$

20. The electric field of an electromagnetic wave has the form  $\vec{E} = E_0 \cos(\omega t - kz) \hat{i}$ . At  $t = 0$ , a test particle of charge  $q$  is at  $z = 0$ , and has velocity  $\vec{v} = 0.5c\hat{k}$ , where  $c$  is the speed of light. The total instantaneous force on the particle is

- (a)  $\frac{qE_0}{2} \hat{i}$  (b)  $\frac{qE_0}{\sqrt{2}} (\hat{i} + \hat{j})$   
 (c)  $\frac{qE_0}{2} (\hat{i} - \hat{k})$  (d) Zero

21. The *rms* velocity of molecules of oxygen gas is given by  $v$  at some temperature  $T$ . The molecules of another gas have the same *rms* velocity at temperature  $\frac{T}{16}$ . The second gas is

- (a) Hydrogen (b) Helium  
 (c) Nitrogen (d) Neon

22. A system undergoes a thermodynamic transformation from state  $S_1$  to state  $S_2$  via two different paths 1 and 2. The heat absorbed and work done along path 1 are  $50 J$  and  $30 J$ , respectively. If the heat absorbed along path 2 is  $30 J$ , the work done along path 2 is

- (a) Zero (b)  $10 J$   
 (c)  $20 J$  (d)  $30 J$

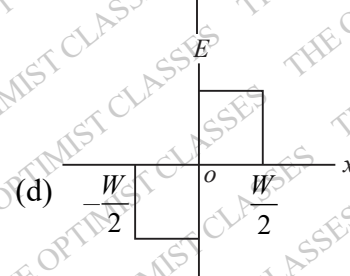
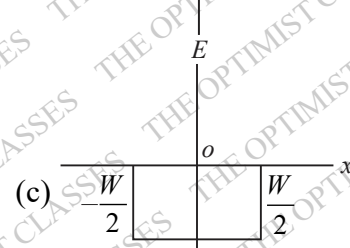
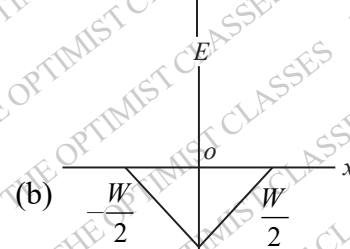
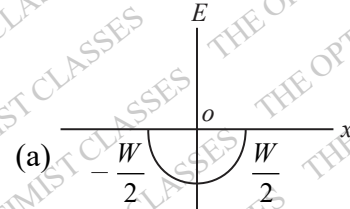
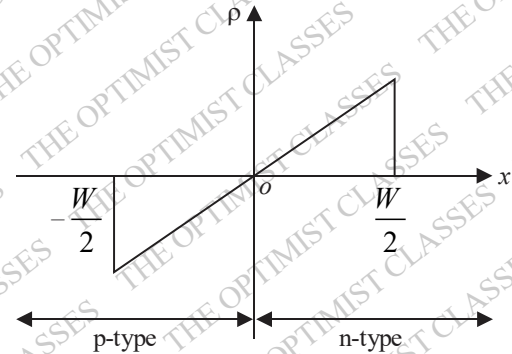
23. The condition for maxima in the interference of two waves  $Ae^{i\left(\frac{k_0}{2}(\sqrt{3}x+y)-\omega t\right)}$  and  $Ae^{i\left(\frac{k_0}{\sqrt{2}}(x+y)-\omega t\right)}$  is given in terms of the wavelength  $\lambda$  and  $m$ , an integer, by

- (a)  $(\sqrt{3} - \sqrt{2})x + (1 - \sqrt{2})y = 2m\lambda$   
 (b)  $(\sqrt{3} + \sqrt{2})x + (1 - \sqrt{2})y = 2m\lambda$

(c)  $(\sqrt{3} - \sqrt{2})x - (1 - \sqrt{2})y = m\lambda$

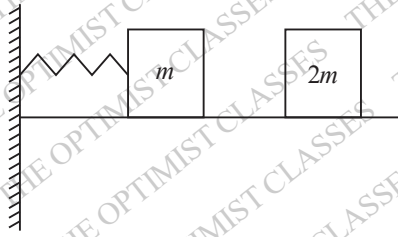
(d)  $(\sqrt{3} - \sqrt{2})x + (1 - \sqrt{2})y = (2m + 1)\lambda$

24. A semiconductor  $pn$  junction at thermal equilibrium has the space charge density  $\rho(x)$  profile as shown in the figure. The figure that best depicts the variation of the electric field  $E$  with  $x$  is ( $W$  denotes the width of the depletion layer)



25. A mass  $m$  is connected to a massless spring of spring constant  $k$ , which is fixed to a wall. Another mass  $2m$ , having kinetic energy  $E$ , collides collinearly with the mass  $m$  completely inelastically (see figure). The entire set up placed on a friction-

less floor. The maximum compression of the spring is



(a)  $\sqrt{\frac{4E}{3k}}$

(b)  $\sqrt{\frac{E}{3k}}$

(c)  $\sqrt{\frac{E}{5k}}$

(d)  $\sqrt{\frac{E}{7k}}$

26. A linearly polarized light falls on a quarter wave plate and the emerging light is found to be elliptically polarized. The angle between the fast axis of the quarter wave plate and the plane of polarization of the incident light, can be

- (a)  $30^\circ$  (b)  $45^\circ$   
(c)  $90^\circ$  (d)  $180^\circ$

27. The expression for the magnetic field that induces the electric field

$$\vec{E} = K(y\hat{i} + 3z\hat{j} + 4y\hat{k})\cos(\omega t) \text{ is}$$

(a)  $-\frac{K}{\omega}(\hat{i} + y\hat{j} - z\hat{k})\sin(\omega t)$

(b)  $-\frac{K}{\omega}(\hat{i} + y\hat{j} + z\hat{k})\sin(\omega t)$

(c)  $-\frac{K}{\omega}(\hat{i} - y\hat{j} + z\hat{k})\sin(\omega t)$

(d)  $-\frac{K}{\omega}(\hat{i} + y\hat{j} + z\hat{k})\sin(\omega t)$

28. In the Fourier series expansion of two functions

$$f_1(t) = 4t^2 + 3 \text{ and } f_2(t) = 6t^3 + 7t \text{ in the interval } -\frac{T}{2} \text{ to } +\frac{T}{2}, \text{ the Fourier coefficients } a_n \text{ and } b_n$$

( $a_n$  and  $b_n$  are coefficients of  $\cos(n\omega t)$  and  $\sin(n\omega t)$ , respectively) satisfy

- (a)  $a_n = 0$  and  $b_n \neq 0$  for  $f_1(t)$ ;  $a_n \neq 0$  and

$b_n = 0$  for  $f_2(t)$

- (b)  $a_n \neq 0$  and  $b_n = 0$  for  $f_1(t)$ ;  $a_n = 0$  and

$b_n \neq 0$  for  $f_2(t)$

- (c)  $a_n \neq 0$  and  $b_n \neq 0$  for  $f_1(t)$ ;  $a_n = 0$  and

$b_n \neq 0$  for  $f_2(t)$

- (d)  $a_n = 0$  and  $b_n \neq 0$  for  $f_1(t)$ ;  $a_n \neq 0$  and

$b_n \neq 0$  for  $f_2(t)$

29. A thin circular disc lying in the xy-plane has a surface mass density  $\sigma$ , given by

$$\sigma(r) = \begin{cases} \sigma_0 \left(1 - \frac{r^2}{R^2}\right) & \text{if } r \leq R \\ 0 & \text{if } r > R \end{cases}$$

where  $r$  is the distance from its center. Its moment of inertia about the z-axis, passing through its center is

(a)  $\frac{\sigma_0 R^4}{4}$  (b)  $\frac{\pi \sigma_0 R^4}{6}$

(c)  $\sigma_0 R^4$  (d)  $2\pi \sigma_0 R^4$

30. The radial component of acceleration in plane polar coordinates is given by

(a)  $\frac{d^2 r}{dt^2}$  (d)  $\frac{d^2 r}{dt^2} - r \left(\frac{d\theta}{dt}\right)^2$

(c)  $\frac{d^2 r}{dt^2} + r \left(\frac{d\theta}{dt}\right)^2$  (d)  $2 \frac{dr}{dt} \frac{d\theta}{dt} + r \frac{d^2 \theta}{dt^2}$

31. A gaseous system, enclosed in an adiabatic container, is in equilibrium at pressure  $P_1$  and volume  $V_1$ . Work is done on the system in a quasi-static manner due to which the pressure and volume changes to  $P_2$  and  $V_2$ , respectively, in the final equilibrium state. At every instant, the pressure and volume obey the condition  $PV^\gamma = C$ , where

$$\gamma = \frac{C_p}{C_v} \text{ and } C \text{ is a constant. If the work done is zero, then identify the correct statement(s).}$$



(a)  $P_2 V_2 = P_1 V_1$  (b)  $P_2 V_2 = \gamma P_1 V_1$

(d)  $P_2 V_2 = (\gamma + 1) P_1 V_1$  (d)  $P_2 V_2 = (\gamma - 1) P_1 V_1$

32. An isolated ideal gas is kept at a pressure  $P_1$  and volume  $V_1$ . The gas undergoes free expansion and attains a pressure  $P_2$  and volume  $V_2$ . Identify the

correct statement(s).  $\left( \gamma = \frac{C_p}{C_v} \right)$

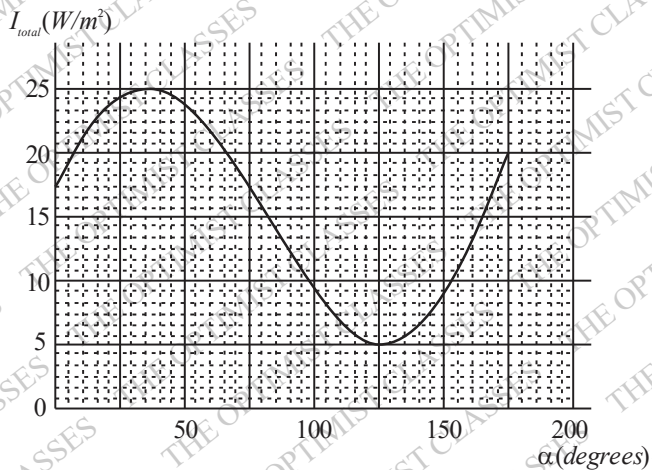
(a) This is an adiabatic process

(b)  $P_1 V_1 = P_2 V_2$

(c)  $P_1 V_1^\gamma = P_2 V_2^\gamma$

(d) This is an isobaric process

33. A beam of light traveling horizontally consists of an unpolarized component with intensity  $I_0$  and a polarized component with intensity  $I_p$ . The plane of polarization is oriented at an angle  $\theta$  with respect to the vertical. The figure shows the total intensity  $I_{\text{total}}$  after the light passes through a polarizer as a function of the angle  $\alpha$ , that the axis of the polarizer makes with respect to the vertical. Identify the correct statement(s).



(a)  $\theta = 125^\circ$

(b)  $I_p = 5 \text{ W/m}^2$

(c)  $I_0 = 17.5 \text{ W/m}^2$

(d)  $I_0 = 10 \text{ W/m}^2$ ;  $I_p = 20 \text{ W/m}^2$

34.

Consider the following differential equation that describes the oscillations of a physical system:

$$\alpha \frac{d^2 y}{dt^2} + \beta \frac{dy}{dt} + \gamma y = 0$$

If  $\alpha$  and  $\beta$  are held fixed, and  $\gamma$  is increased, then,

(a) the frequency of oscillations increases

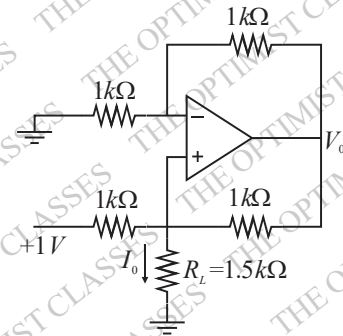
(b) the oscillations decay faster

(c) the frequency of oscillations decreases

(d) the oscillations decay slower

35.

For the given circuit, identify the correct statement(s).



(a)  $I_0 = 1 \text{ mA}$

(b)  $V_o = 3 \text{ V}$

(c) If  $R_L$  is doubled,  $I_0$  will change to  $0.5 \text{ mA}$

(d) If  $R_L$  is doubled,  $V_o$  will change to  $6 \text{ V}$

36.

A Carnot engine operates between two temperatures,  $T_L = 100 \text{ K}$  and  $T_H = 150 \text{ K}$ . Each cycle of the engine lasts for 0.5 seconds during which the power delivered is  $500 \text{ J/second}$ . Let  $Q_H$  be the corresponding heat absorbed by the engine and  $Q_L$  be the heat lost. Identify the correct statement(s).

(a)  $Q_H = 750 \text{ J}$

(b)  $\frac{Q_H}{Q_L} \leq \frac{2}{3}$

(c) The change in entropy of the engine and the hot bath in a cycle is  $5 \text{ J/K}$

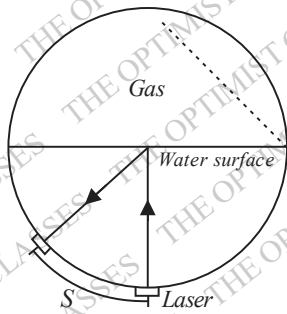


(d) The change in entropy of the engine in 0.5 seconds is zero

37. A time independent conservative force  $\vec{F}$  has the form  $\vec{F} = 3y\hat{i} + f(x, y)\hat{j}$ . Its magnitude at  $x = y = 0$  is 8. The allowed form(s) of  $f(x, y)$  is(are)

- (a)  $3x + 8$  (b)  $2x + 8(y-1)^2$   
(c)  $3x + 8e^{-x^2}$  (d)  $2x + 8\cos y$

38. The figure shows the cross-section of a hollow cylindrical tank,  $2.2m$  in diameter, which is half filled with water (refractive index of 1.33). The space above the water is filled with a gas of unknown refractive index. A small laser moves along the bottom surface and aims a light beam towards the center (see figure). When the laser moves a distance of  $S = 1.09m$  or beyond from the lowest point in the water, no light enters the gas. Identify the correct statement (s) (speed of light is  $3 \times 10^8 m/s$ ).



- (a) The refractive index of the gas is 1.05  
(b) The time taken for the light beam to travel from the laser to the rim of the tank when  $S < 1.09m$  is  $8.9ns$   
(c) The time taken for the light beam to travel from the laser to the rim of the tank when  $S > 1.09m$  is  $9.7ns$   
(d) The critical angle for the water-gas interface is  $56.77^\circ$
39. Identify the correct statement(s) regarding nuclei
- (a) The uncertainty in the momentum of a proton in a nucleus is roughly  $10^5$  times the uncertainty in the momentum of the electron in the ground state

of Hydrogen atom

- (b) The volume of a nucleus grows linearly with the number of nucleons in it  
(c) The energy of  $\gamma$  rays due to de-excitation of a nucleus can be of the order of  $MeV$   
(d)  $^{56}Fe$  is the most stable nucleus

40. A particle of mass  $m$  is in an infinite square well potential of length  $L$ . It is in a superposed state of the first two energy eigenstates, as given by

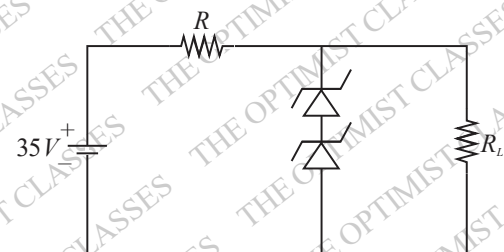
$$\psi(x) = \frac{1}{3}\psi_{n=1}(x) + \sqrt{\frac{2}{3}}\psi_{n=2}(x).$$

Identify the correct statement(s).  $h$  is Planck's constant.

- (a)  $\langle p \rangle = 0$  (b)  $\Delta p = \frac{\sqrt{3}h}{2L}$   
(c)  $\langle E \rangle = \frac{3h^2}{8mL^2}$  (d)  $\Delta x = 0$

41. One of the roots of the equation,  $z^6 - 3z^4 - 16 = 0$  is given by  $z_1 = 2$ . The value of the product of the other five roots is \_\_\_\_.

42. The following Zener voltage regulator circuit is used to obtain  $20V$  regulated output at load resistance  $R_L$  from a  $35V$  dc power supply. Zener diodes are rated at  $5W$  and  $10V$ . The value of the resistance  $R$  is \_\_\_\_  $\Omega$ .



43. A small conducting square loop of side  $l$  is placed inside a concentric large conducting square loop of side  $L$  ( $L \gg l$ ). The value of mutual inductance of the system is expressed as  $\frac{n\mu_0 l^2}{\pi L}$ . The value of  $n$  is \_\_\_\_ (Round off to two decimal places).

44. Consider  $N_1$  number of ideal gas particles en-

closed in a volume  $V_1$ . If the volume is changed to  $V_2$  and the number of particles is reduced by half, the mean free path becomes four times of its initial value. The ratio  $\frac{V_1}{V_2}$  is \_\_\_\_ (Round off to one decimal place).

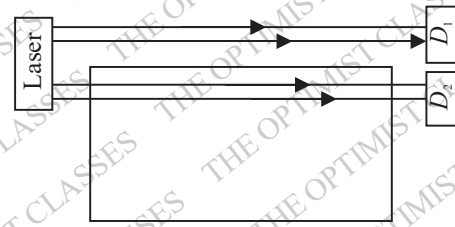
45. A particle is moving with a velocity  $0.8c\hat{i}$  ( $c$  is the speed of light) in an inertial frame  $S_1$ . Frame  $S_2$  is moving with a velocity  $0.8c\hat{i}$  with respect to  $S_1$ . Let  $E_1$  and  $E_2$  be the respective energies of the particle in the two frames. Then,  $\frac{E_2}{E_1}$  is \_\_\_\_.

(Round off to two decimal places).

46. At some temperature  $T$ , two metals  $A$  and  $B$ , have Fermi energies  $\epsilon_A$  and  $\epsilon_B$ , respectively. The free electron density of  $A$  is 64 times that of  $B$ . The ratio  $\frac{\epsilon_A}{\epsilon_B}$  is \_\_\_\_.

47. A crystal has monoclinic structure, with lattice parameters,  $a = 5.14 \text{ \AA}$ ,  $b = 5.20 \text{ \AA}$ ,  $c = 5.30 \text{ \AA}$  and  $\beta = 99^\circ$ . It undergoes a phase transition to tetragonal structure with lattice parameters,  $a = 5.09 \text{ \AA}$  and  $c = 5.27 \text{ \AA}$ . The fractional change in the volume  $\left| \frac{\Delta V}{V} \right|$  of the crystal due to the transition is \_\_\_\_ (Round off to two decimal places).

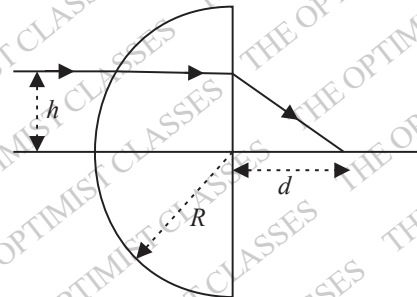
48. A laser beam shines along a block of transparent material of length  $2.5 \text{ m}$ . Part of the beam goes to the detector  $D_1$  while the other part travels through the block and then hits the detector  $D_2$ . The time delay between the arrivals of two light beams is inferred to be  $6.25 \text{ ns}$ . The speed of light  $c = 3 \times 10^8 \text{ m/s}$ . The refractive index of the block is \_\_\_\_ (Round off to two decimal places).



49. An ideal black body at temperature  $T$ , emits radiation of energy density  $u$ . The corresponding value for a material at temperature  $\frac{T}{2}$  is  $\frac{u}{256}$ . Its emissivity is \_\_\_\_ (Round off to three decimal places).

50. A particle with positive charge  $10^{-3} \text{ C}$  and mass  $0.2 \text{ kg}$  is thrown upwards from the ground at an angle  $45^\circ$  with the horizontal with a speed of  $5 \text{ m/s}$ . The projectile moves through a horizontal electric field of  $10 \text{ V/m}$ , which is in the same direction as the horizontal component of the initial velocity of the particle. The acceleration due to gravity is  $10 \frac{\text{m}}{\text{s}^2}$ . The range is \_\_\_\_  $\text{m}$ . (Round off to three decimal places).

51. Consider a hemispherical glass lens (refractive index is 1.5) having radius of curvature  $R = 12 \text{ cm}$  for the curved surface. An incoming ray, parallel to the optical axis, is incident on the curved surface at a height  $h = 1 \text{ cm}$  above the optical axis, as shown in the figure. The distance  $d$  (from the flat surface of the lens) at which the ray crosses the optical axis is \_\_\_\_  $\text{cm}$  (Round off to two decimal places).



52. Twenty non-interacting spin  $\frac{1}{2}$  particles are trapped in a three-dimensional simple harmonic

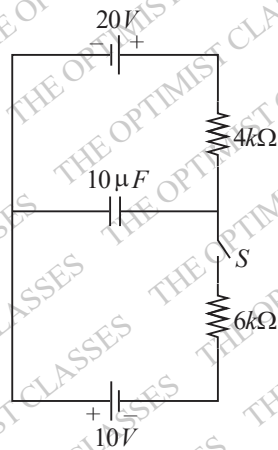


oscillator potential of frequency  $\omega$ . The ground state energy of the system, in units of  $\hbar\omega$ , is \_\_\_\_.

53. A thin film of alcohol is spread over a surface. When light from a tunable source is incident normally, the intensity of reflected light at the detector is maximum for  $\lambda = 640\text{nm}$  and minimum for  $\lambda = 512\text{nm}$ . Taking the refractive index of alcohol to be 1.36 for both the given wavelength, the minimum thickness of the film would be \_\_\_\_ nm (Round off to two decimal places).

54. For the Boolean expression  $Y = ABC + \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C}$ , the number of combinations for which the output  $Y = 1$  is \_\_\_\_.

55. An RC circuit is connected to two dc power supplies, as shown in the figure. With switch  $S$  open, the capacitor is fully charged.  $S$  is then closed at time  $t = 0$ . The voltage across the capacitor at  $t = 2.4$  milliseconds is \_\_\_\_ V. (Round off one decimal places)



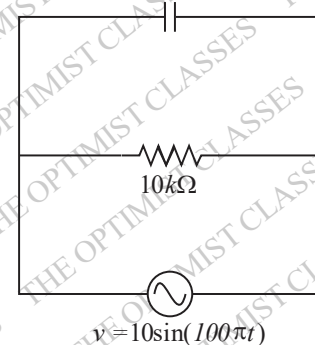
56. A current  $I$  is uniformly distributed across a long straight nonmagnetic wire ( $\mu_r = 1$ ) of circular cross-section with radius  $a$ . Two points  $P$  and  $Q$  are at distance  $\frac{a}{3}$  and  $9a$ , respectively, from the axis of the wire. The ratio of the magnetic field at points  $P$  and  $Q$  is \_\_\_\_.

57. A particle  $A$  of mass  $m$  is moving with a velocity  $\hat{v}_i$ , and collides elastically with a particle  $B$ , of mass  $2m$ .  $B$  is initially at rest. After collision,  $A$

moves with a velocity  $\hat{v}_A$ . If  $\hat{v}_B$  is the final speed of  $B$ , then  $v_A^2 = kv_B^2$ . The value of  $k$  is \_\_\_\_.

58. In an X-ray diffraction experiment with  $\text{Cu}$  crystals having lattice parameter  $3.61\text{\AA}$ , X-rays of wavelength of  $0.090\text{nm}$  are incident on the family of planes  $\{110\}$ . The highest order present in the diffraction pattern is \_\_\_\_.

59. A parallel plate capacitor having plate area of  $50\text{cm}^2$  and separation of  $0.1\text{mm}$  is completely filled with a dielectric (dielectric constant  $K = 10$ ). The capacitor is connected to a  $10\text{k}\Omega$  resistance and an alternating voltage  $v = 10\sin(100\pi t)$ , as shown in the figure. The switch  $S$  is initially open and then closed at  $t = 0$ . The ratio of the displacement current in the capacitor, to the current in the resistance, at time  $t = \frac{2}{\pi}$  seconds is \_\_\_\_.



60. The wavelength of characteristic  $K_\alpha$  X-ray photons from  $M_0$  (atomic number 42) is \_\_\_\_ Å. (Round off to one decimal place). (speed of light is  $3 \times 10^8\text{m/s}$ ; Rydberg constant  $R = 1.09 \times 10^7/\text{m}$ ).

# ANSWER KEY

- |                        |                    |
|------------------------|--------------------|
| 1. (b)                 | 2. (a)             |
| 3. (d)                 | 4. (a)             |
| 5. (a)                 | 6. (b)             |
| 7. (a)                 | 8. (d)             |
| 9. (c)                 | 10. (c)            |
| 11. (a)                | 12. (a)            |
| 13. (a)                | 14. (a)            |
| 15. (a)                | 16. (d)            |
| 17. (a)                | 18. (a)            |
| 19. (a)                | 20. (a)            |
| 21. (a)                | 22. (b)            |
| 23. (a)                | 24. (a)            |
| 25. (a)                | 26. (a)            |
| 27. (a)                | 28. (b)            |
| 29. (b)                | 30. (b)            |
| 31. (a)                | 32. (a,b)          |
| 33. (d)                | 34. (a)            |
| 35. (a,b,d)            | 36. (a,c,d)        |
| 37. (a,c)              | 38. (a,c,d)        |
| 39. (a,b,c,d)          | 40. (a,b,c)        |
| 41. (-8)               | 42. (30)           |
| 43. (2.80 to 2.85)     | 44. 0.5            |
| 45. (1.64 to 1.68)     | 46. (16)           |
| 47. (0.02 to 0.03)     | 48. (1.73 to 1.77) |
| 49. (0.060 to 0.065)   |                    |
| 50. (2.510 to 2.515)   |                    |
| 51. (15.81 to 16.01)   |                    |
| 52. (60)               |                    |
| 53. (470.56 to 470.62) |                    |
| 54. (4)                |                    |
| 55. (18.5 to 19)       |                    |
| 56. (3)                |                    |
| 57. (1)                |                    |
| 58. (5)                |                    |
| 59. (0.036 to 0.040)   |                    |
| 60. (0.6 to 0.8)       |                    |