# THE OPTIMIST CLASSES
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MANOJ KUMAR SINGH


ABHAY


PAWAN


SATYAM


SOUMIL GIRISH SAHU


BHOOMIJA


AKSHIT AGGARWAL


SHIKHAR CHAMOLI


RAVI SINGH ADHIKARI


GAURAV JHA


SWAPNIL JOSHI


LOKESH BHATT


GOPESH VISHVAKARMA


VAIBHAV


SHASWAT CHAMOLI

# CSIR-NET-JRF RESULTS 2022


**ANNU**
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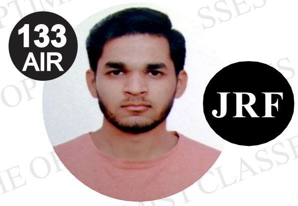



**ALANKAR**
**UP15000162**


**SAHIL RANA**
**HR09000108**


**JAYESTHI**
**RJ11000161**


**DASRATH**
**RJ06000682**


**VIVEK**
**UK01000439**


**UZAIR AHMED**
**UP02000246**


**SURYA PRATAP SINGH**
**RJ06000232**


**HIMANSHU**
**UP10000095**


**CHANDAN**
**RJ09000159**


**SAIKHOM JOHNSON**
**MN01000196**


**AJAY SAINI**
**RJ06001744**


**VIKAS YADAV**
**RJ06001102**


**JYOTSNA KOHLI**
**UK02000262**


**SHYAM SUNDAR**
**RJ06000615**

# THE OPTIMIST CLASSES

AN INSTITUTE FOR NET-JRF/GATE/IIT-JAM/JEST/TIFR/M.Sc ENTRANCE EXAMS

**CONTACT: 9871044043**


## IIT-JAM PAPER 2022

1. The equation $z^2+\overline{z}^2=4$ in the complex plane (where $\overline{z}$ is the complex conjugate of $z$) represents
(a) Ellipse (b) Hyperbola
(c) Circle of radius 2 (d) Circle of radius 4

2. A rocket $(S')$ moves at a speed $\frac{c}{2}m/s$ along the positive $x$-axis, where c is the speed of light. When it crosses the origin,the clocks attached to the rocket and the one with a stationary observer ($S$) located at $x=0$ are both set to zero. It $S$ observes an event at ($x$, $t$) the same event cours in the $S'$ frame at
(a) $x'=\frac{2}{\sqrt{3}}\left(x-\frac{ct}{2}\right)$ and $t'=\frac{2}{\sqrt{3}}\left(t-\frac{x}{2c}\right)$
(b) $x'=\frac{2}{\sqrt{3}}\left(x+\frac{ct}{2}\right)$ and $t'=\frac{2}{\sqrt{3}}\left(t-\frac{x}{2c}\right)$
(c) $x'=\frac{2}{\sqrt{3}}\left(x-\frac{ct}{2}\right)$ and $t'=\frac{2}{\sqrt{3}}\left(t+\frac{x}{2c}\right)$
(d) $x'=\frac{2}{\sqrt{3}}\left(x+\frac{ct}{2}\right)$ and $t'=\frac{2}{\sqrt{3}}\left(t+\frac{x}{2c}\right)$

3. Consider a classical ideal gas of $N$ molecules in equilibrium at temperature $T$. Each molecule has two energy levels, $-\varepsilon$ and $\varepsilon$. The mean energy of the gas is
(a) 0 (b) $N\varepsilon\tanh\left(\frac{\varepsilon}{k_BT}\right)$
(c) $-N\varepsilon\tanh\left(\frac{\varepsilon}{k_BT}\right)$ (d) $\frac{\varepsilon}{2}$

4. At temperature T, let $\beta$ amd $\kappa$ denote the volume expansivity and isothermal compressibility of a gas, respectively. Then $\frac{\beta}{\kappa}$ is equal to
(a) $\left(\frac{\partial P}{\partial T}\right)_V$ (b) $\left(\frac{\partial P}{\partial V}\right)_T$
(c) $\left(\frac{\partial T}{\partial P}\right)_V$ (d) $\left(\frac{\partial T}{\partial V}\right)_P$

5. The resultant of the binary subtraction 1110101 – 0011110 is
(a) 1001111 (b) 1010111
(c) 1010011 (d) 1010001

6. Consider a particle trapped in a three-dimensional potential well such that $U$ ($x$, $y$, $z$) = 0 for $0\le x\le a$, $0\le y\le a$, $0\le z\le a$ and $U(x,y,z)=\infty$ everywhere else. The degeneracy of the 5th excited state is
(a) 1 (b) 3
(c) 6 (d) 9

7. A particle of mass $m$ and angular momentum $L$ moves in space where its potential energy is $U(r)=kr^2\,(k>0)$ and $r$ is the radial coordinate. If the particle moves in a circular orbit, then the radius of the orbit is

(a) $\left(\frac{L^2}{mk}\right)^{\frac{1}{4}}$ (b) $\left(\frac{L^2}{2mk}\right)^{\frac{1}{4}}$

(c) $\left(\frac{2L^2}{mk}\right)^{\frac{1}{4}}$ (d) $\left(\frac{4L^2}{mk}\right)^{\frac{1}{4}}$

8. Consider a two-dimensional force field

$$\vec{F}(x,y) = \left(5x^2 + ay^2 + bxy\right)\hat{x} + \left(4x^2 + 4xy + y^2\right)\hat{y}$$

If the force field is conservative, then the values of $a$ and $b$ are

(a) $a = 2$ and $b = 4$

(b) $a = 2$ and $b = 8$

(c) $a = 4$ and $b = 2$

(d) $a = 8$ and $b = 2$

9. Consider an electrostatic field $\vec{E}$ in a region of space. Identify the **INCORRECT** statement.

(a) The work done in moving a charge a closed path inside the region is zero

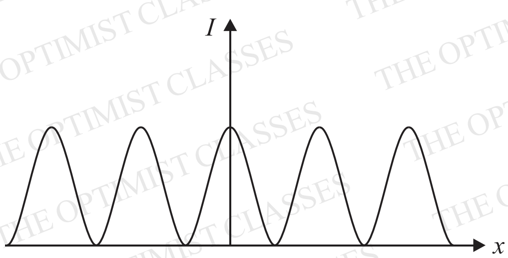
(b) The curl of $\vec{E}$ is zero

(c) The field can be expressed as the gradient of a scalar potential

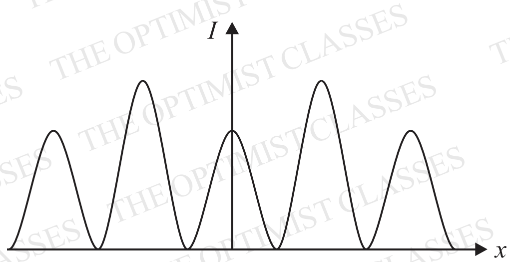
(d) The potential differenece between any two points in the region is always zero

10. Which one of the following figures correctly depicts the intensity distribution for Fraunhofer diffraction due to a single slit? Here, $x$ denotes the distance from the centre of the central fringe and $I$ denotes the intensity.

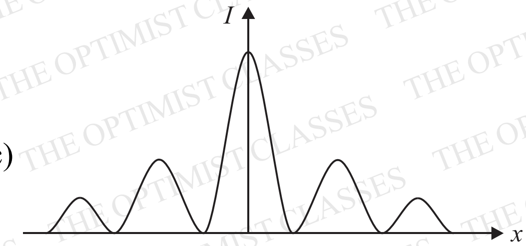
(a)



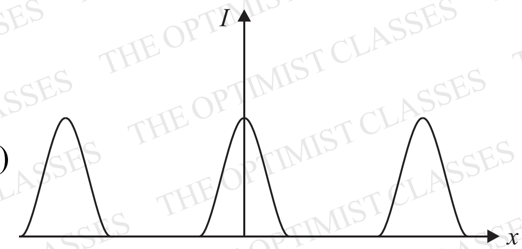

(b)




(c)




(d)




11. The function $f(x) = e^{\sin x}$ is expanded as a Taylor series in $x$, around $x = 0$, in the form $f(x) = \sum_{n=0}^{\infty} a_n x^n$. The value of $a_0 + a_1 + a_2$ is

(a) 0 (b) $\frac{3}{2}$

(c) $\frac{5}{2}$ (d) 5

12. Consider a unit circle $c$ in the $xy$-plane, centered at the origin. The value of the integral $\oint\left[(\sin x - y)dx - (\sin y - x)dy\right]$ over the circle $C$, traversed anticlockwise, is

(a) 0 (b) $2\pi$

(c) $3\pi$ (d) $4\pi$

13. The current through a series $RL$ circuit, subjected to a constant emf $\varepsilon$, obeys $L\frac{di}{dt} + iR = \varepsilon$.

Let $L = 1mH$, $R = 1k\Omega$ and $\varepsilon = 1V$. The initial condition is i(0) = 0. At $t = 1\mu s$, the current in $mA$ is

(a) $1 - 2e^{-2}$ (b) $1 - 2e^{-1}$

(c) $1 - e^{-1}$ (d) $2 - 2e^{-1}$

14. An ideal gas in equilibrium at temperature $T$ expands isothermally to twice its initial volume. If $\Delta S, \Delta U$ and $\Delta F$ denote the changes in its entropy, internal energy and Helmholtz free energy respectively, then

(a) $\Delta S < 0, \Delta U > 0, \Delta F < 0$

(b) $\Delta S > 0, \Delta U = 0, \Delta F < 0$

(c) $\Delta S < 0, \Delta U = 0, \Delta F > 0$

(d) $\Delta S > 0, \Delta U > 0, \Delta F = 0$

15. In a dilute gas, the number of molecules with free path length $\geq x$ is given by $N(x) = N_0 e^{-x/\lambda}$, where $N_0$ is the total number of molecules and $\lambda$ is the mean free path. The fraction of molecules with free path length between $\lambda$ and $2\lambda$ is

(a) $\frac{1}{e}$ (b) $\frac{e}{e-1}$

(c) $\frac{e^2}{e-1}$ (d) $\frac{e-1}{e^2}$

16. Consider a quantum particle trapped in a one-dimensional potential well in the region $\left[-\frac{L}{2} < x < \frac{L}{2}\right]$, with infinitely high barriers at $x = -\frac{L}{2}$ and $x = \frac{L}{2}$. The stationary wave function for the ground state is $\psi(x) = \sqrt{\frac{2}{L}}\cos\left(\frac{\pi x}{L}\right)$. The uncertainties in momentum and position satisfy

(a) $\Delta p = \frac{\pi\hbar}{L}$ and $\Delta x = 0$

(b) $\Delta p = \frac{2\pi\hbar}{L}$ and $0 < \Delta x < \frac{L}{2\sqrt{3}}$

(c) $\Delta p = \frac{\pi\hbar}{L}$ and $\Delta x > \frac{L}{2\sqrt{3}}$

(d) $\Delta p = 0$ and $\Delta x = \frac{L}{2}$

17. Consider a particle of mass $m$ moving in a plane with a constant radial speed $\dot{r}$ and a constant angular speed $\dot{\theta}$. The acceleration of the particle in $(r,\theta)$ coordinate is

(a) $2r\dot{\theta}^2\hat{r} - \dot{r}\dot{\theta}\hat{\theta}$ (b) $-r\dot{\theta}^2\hat{r} - 2\dot{r}\dot{\theta}\hat{\theta}$

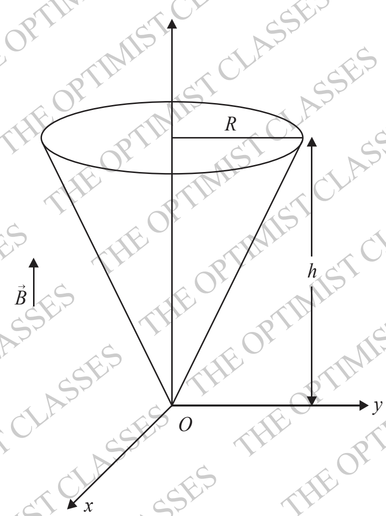
(c) $\ddot{r}\hat{r} + r\ddot{\theta}\hat{\theta}$ (d) $\ddot{r}\theta\hat{r} + r\ddot{\theta}\hat{\theta}$

18. A planet of mass $m$ moves in an elliptical orbit. Its maximum and minimum distances from the Sun are $R$ and $r$, respectively. Let $G$ denote the universal gravitational constant, and $M$ the mass of the Sun. Assuming $M >> m$, the angular momentum of the planet with respect to the center of the Sun is

(a) $m\sqrt{\frac{2GMRr}{(R+r)}}$ (b) $m\sqrt{\frac{GMRr}{2(R+r)}}$

(c) $m\sqrt{\frac{GMRr}{(R+r)}}$ (d) $2m\sqrt{\frac{2GMRr}{(R+r)}}$

19. Consider a conical region of height $h$ and base radius $R$ with its vertex at the origin. Let the outward normal to its base be along the positive $z$-axis, as shown in the figure. A uniform magnetic field, $\vec{B} = B_0\hat{z}$ exists everywhere. Then the magnetic flux through the base $(\Phi_b)$ and that through the curved surface of the cone $(\Phi_c)$ are



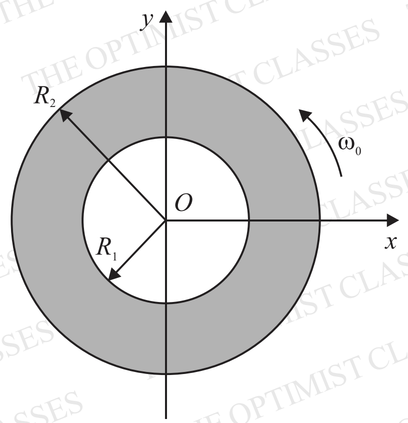

(a) $\Phi_b = B_0\pi R^2, \Phi_c = 0$

(b) $\Phi_b = -\frac{1}{2}B_0\pi R^2, \Phi_c = \frac{1}{2}B_0\pi R^2$

(c) $\Phi_b = 0, \Phi_c = -B_0\pi R^2$

(d) $\Phi_b = B_0\pi R^2, \Phi_c = -B_0\pi R^2$

20. Consider a thin annular sheet, lying on the $xy$-plane, with $R_1$ and $R_2$ as its inner and outer radii, respectively. If the sheet carries a uniform surface-charge density $\sigma$ and spins about the origin $O$ with a constant angular velocity $\vec{\omega} = \omega_0 \hat{z}$ then, the total current flow on the sheet is




(a) $\frac{2\pi\sigma\omega_0\left(R_2^3 - R_1^3\right)}{3}$

(b) $\sigma\omega_0\left(R_2^3 - R_1^3\right)$

(c) $\frac{\pi\sigma\omega_0\left(R_2^3 - R_1^3\right)}{3}$

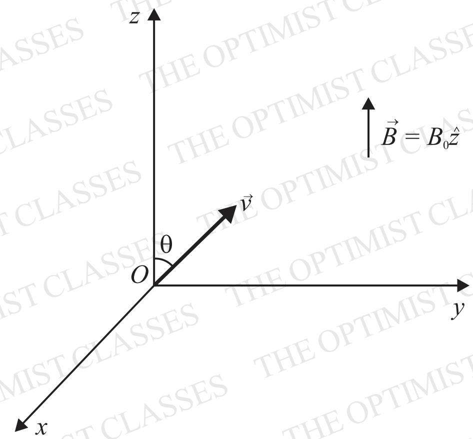
(d) $\frac{2\pi\sigma\omega_0\left(R_2 - R_1\right)^3}{3}$

21. A radioactive nucleus has a decay constant $\lambda$ and its radioactive daughter nucleus has a decay constant $10\lambda$. At time $t=0$, $N_0$ is the number of parent nuclei and there are no daughter nuclei present. $N_1(t)$ and $N_2(t)$ are the number of parent and daughter nuclei present at time $t$, respectively. The ratio $\frac{N_2(t)}{N_1(t)}$ is

(a) $\frac{1}{9}\left[1-e^{-9\lambda t}\right]$ (b) $\frac{1}{10}\left[1-e^{-10\lambda t}\right]$

(c) $\left[1-e^{-10\lambda t}\right]$ (d) $\left[1-e^{-9\lambda t}\right]$

22. A uniform magnetic field $\vec{B} = B_0\hat{z}$, where $B_0 > 0$ exists as shown in the figure. A charged particle of mass m and charge $q(q>0)$ is released at the origin, in the $yz$-plane, with a velocity $\vec{v}$ directed at an angle $\theta = 45°$ with respect to the positive $z$-axis. Ignoring gravity, which one of the following is **TRUE**.




(a) The initial acceleration $\vec{a} = \frac{qvB_0}{\sqrt{2}m}\hat{x}$

(b) The initial acceleration $\vec{a} = \frac{qvB_0}{\sqrt{2}m}\hat{y}$

(c) The particle moves in a circular path

(d) The particle continues in a straight line with constant speed

23. For an ideal intrinsic semiconductor, the Fermi energy at $0K$

(a) lies at the top of the valence band

(b) lies at the bottom of the conduction band

(c) lies at the center of the bandgap

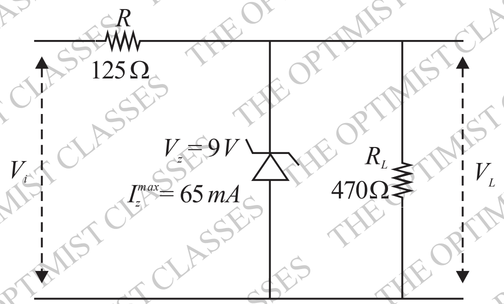
(d) lies midway between center of the bandgap and bottom of the conduction band

24. A circular loop of wire with radius $R$ is centered at the origin of the $xy$-plane. The magnetic field at a point within the loop is, $\vec{B}(\rho,\phi,z,t) = k\rho^3 t^2\hat{z}$, where $k$ is a positive constant of appropriate dimensions. Neglecting the effects of any current induced in the loop, the magnitude of the induced *emf* in the loop at time $t$ is

(a) $\frac{6\pi k t^2 R^5}{5}$ (b) $\frac{5\pi k t^2 R^5}{6}$

(c) $\dfrac{3\pi k t^2 R^5}{2}$ (d) $\dfrac{\pi k t^2 R^5}{2}$

25. For the given circuit, $R = 125\Omega, R_L = 470\Omega$, $V_z = 9V, I_z^{max} = 65mA$. The minimum and maximum values of the input voltage ($V_z^{min}$ and $V_z^{max}$) for which the Zener diode will be in the 'ON' state are



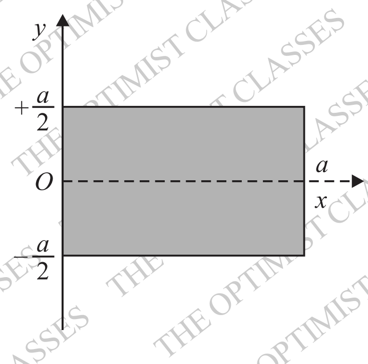

(a) $V_i^{min} = 9.0V$, and $V_i^{max} = 11.4V$

(b) $V_i^{min} = 9.0V$, and $V_i^{max} = 19.5V$

(c) $V_i^{min} = 11.4V$, and $V_i^{max} = 15.5V$

(d) $V_i^{min} = 11.4V$, and $V_i^{max} = 19.5V$

26. A square laminar sheet with side $a$ and mass $M$, has mass per unit area given by $\sigma(x) = \sigma_0\left[1 - \frac{x}{a}\right]$, (see figure). Moment of inertia of the sheet about $y$-axis is




(a) $\dfrac{Ma^2}{2}$ (b) $\dfrac{Ma^2}{4}$

(c) $\dfrac{Ma^2}{6}$ (d) $\dfrac{Ma^2}{12}$

27. A particle is subjected to two simple harmonic motions along the $x$ and $y$ axes, described by $x(t) = a\sin(2\omega t + \pi)$ and $y(t) = 2a\sin(\omega t)$. The resultant motion is given by

(a) $\dfrac{x^2}{a^2} + \dfrac{y^2}{4a^2} = 1$ (b) $x^2 + y^2 = 1$

(c) $y^2 = x^2\left(1 - \dfrac{x^2}{4a^2}\right)$ (d) $x^2 = y^2\left(1 - \dfrac{y^2}{4a^2}\right)$

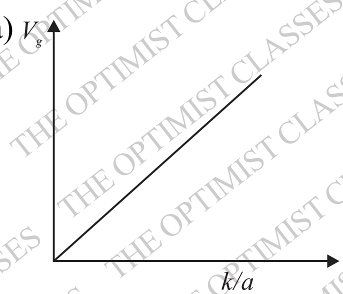
28. For a certain thermodynamic system, the internal energy $U = PV$ and $P$ is proportional to $T^2$. The entropy of the system is proportional to

(a) $UV$ (b) $\sqrt{\dfrac{U}{V}}$

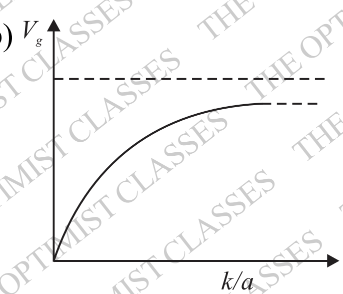
(c) $\sqrt{\dfrac{V}{U}}$ (d) $\sqrt{UV}$

29. The dispersion relation for certain type of waves is given by $\omega = \sqrt{k^2 + a^2}$ where $k$ is the wave vector and $a$ is a constant. Which one of the following sketches represents $v_g$, the group velocity?

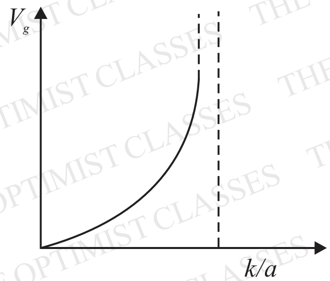
(a)



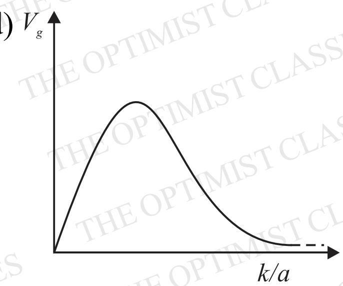

(b)



(c)




(d)




30. Consider a binary number with $m$ digits, where m is an even number. This binary number has alternating 1'$s$ and 0'$s$ with digit 1 in the highest place value. The decimal equivalent of this binary number is

(a) $2^m - 1$  (b) $\frac{(2^m - 1)}{3}$

(c) $\frac{(2^{m+1} - 1)}{3}$  (d) $\frac{2}{3}(2^m - 1)$

31. Consider the 2 × 2 matrix $M = \begin{pmatrix} 0 & a \\ a & b \end{pmatrix}$, where $a, b > 0$. Then,

(a) $M$ is a real symmetric matrix
(b) One of the eigenvalues of $M$ is greater than $b$
(c) One of the eigenvalues of $M$ is negative
(d) Product of eigenvalues of $M$ is $b$

32. In the Compton scattering of electrons, by photons incident with wavelength $\lambda$,

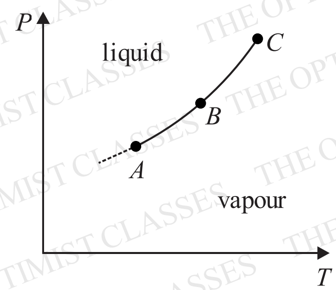
(a) $\frac{\Delta\lambda}{\lambda}$ is independent of $\lambda$

(b) $\frac{\Delta\lambda}{\lambda}$ increases with decreasing $\lambda$

(c) There is no change in photon's wavelength for all angles of deflection of the photon

(d) $\frac{\Delta\lambda}{\lambda}$ increases with increasing angle of deflection of the photon

33. The figure shown a section of the phase boundary separating the vapour (1) and liquid (2) states of water in the $P - T$ plane. Here, $C$ is the critical point $\mu_1, v_1$ and $s_1$ are the chemical potential, specific volume and specific entropy of the vapour phase respectively, while $\mu_2, v_2$ and $s_2$ respectively denote the same for the liquid phase. Then




(a) $\mu_1 = \mu_2$ along $AB$
(b) $v_1 = v_2$ along $AB$
(c) $s_1 = s_2$ along $AB$
(d) $v_1 = v_2$ at the point $C$

34. A particle is executing simple harmonic motion with time period $T$. Let $x$, $v$ and $a$ denote the displacement, velocity and acceleration of the particle, respectively, at time $t$. Then,

(a) $\frac{aT}{x}$ does not change with time

(b) $(aT + 2\pi v)$ does not change with time

(c) $x$ and $v$ are related by an equation of a straight line

(d) $v$ and $a$ are realted by an equation of an ellipse

35. A linearly polarized light beam travels from origin to point $A$ (1, 0, 0). At the point $A$, the light is reflected by a mirror towards point $B$(1, –1, 0). A second mirror located at point $B$ then reflects the light towards point $C$(1, –1, 1). Let $\hat{n}(x, y, z)$ represent the direction of polarization of light at ($x,y,z$).

(a) If $\hat{n}(0,0,0) = \hat{y}$ then $\hat{n}(1,-1,1) = \hat{x}$

(b) If $\hat{n}(0,0,0) = \hat{z}$ then $\hat{n}(1,-1,1) = \hat{y}$

(c) If $\hat{n}(0,0,0) = \hat{y}$ then $\hat{n}(1,-1,1) = \hat{y}$

(d) $\hat{n}(0,0,0) = \hat{z}$ then $\hat{n}(1,-1,1) = \hat{x}$

36. Let $(r,\theta)$ denote the polar coordinates of a particle moving in a plane. If $\hat{r}$ and $\hat{\theta}$ represent the corresponding unit vectors, then

(a) $\frac{d\hat{r}}{d\theta} = \hat{\theta}$ (b) $\frac{d\hat{r}}{dr} = -\hat{\theta}$

(b) $\frac{d\hat{\theta}}{d\theta} = -\hat{r}$ (d) $\frac{d\hat{\theta}}{dr} = \hat{r}$

37. The electric field associated with an electromagnetic radiation is given by $E = a(1+\cos\omega_1 t)\cos\omega_2 t$. Which of the following frequencies are present in the field?

(a) $\omega_1$ (b) $\omega_1 + \omega_2$

(c) $|\omega_1 - \omega_2|$ (d) $\omega_2$

38. A string of length L is stretched between two points $x = 0$ and $x = L$ and the endpoints are rigidly clamped. Which of the following can represent the displacement of the string from the equilibrium position?

(a) $x\cos\left(\frac{\pi x}{L}\right)$ (b) $x\sin\left(\frac{\pi x}{L}\right)$

(c) $x\left(\frac{x}{L}-1\right)$ (d) $x\left(\frac{x}{L}-1\right)^2$

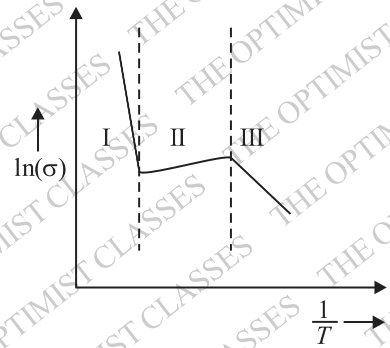
39. The Boolean expression

$Y = \overline{PQ}R + Q\overline{R} + \overline{P}QR + PQR$ simplifies to

(a) $\overline{P}R + Q$ (b) $PR + \overline{Q}$

(c) $P + R$ (d) $Q + R$

40. For an $n$-type silicon, an extrinsic semiconductor, the natural logarithm or normalized conductivity $(\sigma)$ is plotted as a function of inverse temperature. Temperature interval-I corresponds to the intrinsic regime, interval-II corresponds to saturation regime and interval-III corresponds to the freeze-out regime, respectively. Then




(a) the magnitude of the slope of the curve in the temperature interval-I is proportional to the bandgap, $E_g$

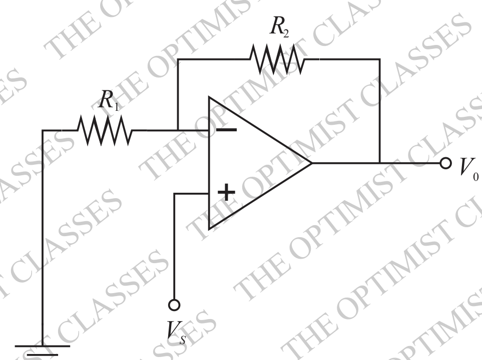
(b) the magnitude of the slope of the curve in the temperature interval-III is proportional to the ionization energy of the donor, $E_d$

(c) in the temperature interval-II, the carrier in the conduction band is equal to the density of donors

(d) in the temperature intervals-III, all the donor levels are ionized

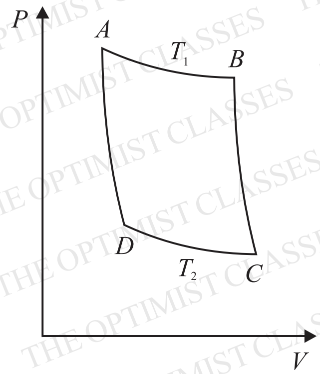
41. The integral $\iint (x^2 + y^2)\,dxdy$ over the area of a disk of radius 2 in the $xy$ plane is ____ $\pi$.

42. For the given operational amplifier circuit $R_1 = 120\Omega, R_2 = 1.5k\Omega$ and $V_S = 0.6V$, then the output current $I_0$ is ____ $mA$.




43. For an ideal gas, $AB$ and $CD$ are two isothermals at temperatures $T_1$ and $T_2$ $(T_1 > T_2)$, respectively. $AD$ and $BC$ represent two adiabatic paths as shown in figure. Let $V_A$, $V_B$, $V_C$ and $V_D$ be the volumes of the gas at $A$, $B$, $C$ and $D$ respectively.

If $\frac{V_C}{V_B} = 2$, then $\frac{V_D}{V_A} =$ ____.




44. A satellite is revolving around the Earth in a closed orbit. The height of the satellite above Earth's surface at perigee and apogee are 2500*km* and 4500*km*, respectively. Consider the radius of the Earth to be 6500*km*. The eccentricity of the satellite's orbit is_____
(Round off to 1 decimal place)

45. Three masses $m_1 = 1$, $m_2 = 2$ and $m_3 = 3$ are located on the *x*-axis such the their center of mass is at $x = 1$. Another mass $m_4 = 4$ is placed at $x_0$ and the new center of mass is at $x = 3$. The value of $x_0$ is________

46. A normal human eye can distinguish two objects separated by 0.35*m* when viewed from a distance of 1.0*km*. The angular resolution of eye is_________ seconds
(Round off to the nearest interger)

47. A rod with a proper length of 3*m* moves along *x*-axis, making an angle of 30° with respect to the *x*-axis. If its speed is $\frac{c}{2} m/s$, where *c* is the speed of light, the change in length due to Lorentz contraction is________ *m*
(Round off to 2 decimal places)
[Use $c = 3 \times 10^8 m/s$]

48. Consider the Bohr model of hydrogen atom. The speed of an electron in the second orbit ($n = 2$) is__________ $\times 10^6 m/s$.
(Round off to 2 decimal places)
[Use $h = 6.63 \times 10^{-34}$ *Js*, $e = 1.6 \times 10^{-19}$ *C*, $\varepsilon_0 = 8.85 \times 10^{-12} C^2 m^2 / N$]

49. Consider a unit circle *C* in the *xy* plane with center at the origin. The line integral of the vector field, $\vec{F}(x, y, z) = -2y\hat{x} - 3z\hat{y} + x\hat{z}$, taken anticlockwise over *C* is________ $\pi$.

50. Consider a *p-n* junction at $T = 300K$. The saturation current density at reverse bias is $-20\mu A / cm^2$. For this device, a current density of magnitude $10\mu A / cm^2$ is realized with a forward bias voltage, $V_F$. The same magnitude of current density can also be realized with a reverse bias voltage, $V_R$. The value of $\left|\frac{V_F}{V_R}\right|$ is________
(Round off to 2 decimal places)

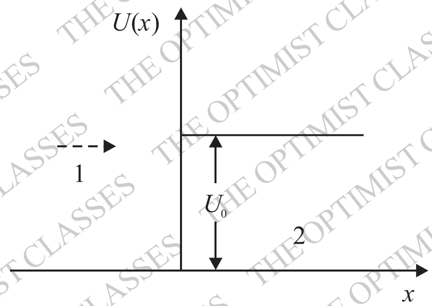
51. Consider the second order ordinary differential equation, $y'' + 4y' + 5y = 0$. If $y(0) = 0$ and $y'(0) = 1$, then the value of $y(\pi / 2)$ is____.
(Round off to 3 decimal places).

52. A box contains a mixture of two different ideal monoatomic gases,1 and 2, in equilibrium at temperature *T*. Both gases are present in equal proportions. The atomic mass for gas 1 is *m*, while the same for gas 2 is 2*m*. If the rms speed of a gas molecule selected at random is $r_{rms} = x\sqrt{\frac{k_B T}{m}}$, then *x* is (Round off to 2 decimal places).

53. A hot body with constant heat 800*J/K* at temperature 925*K* is dropped gently into a vessel containing 1*kg* of water at temperature 300*K* and the combined system is allowed to reach equilibrium. The change in the total entropy $\Delta S$ is _____*J/K*. (Round off to 1 decimal place).
[Take the specific heat capacitiy of water to be 4200*J/kgK*. Neglect any loss of heat to the vessel and air and change in the volume of water.]

54. Consider an electron with mass *m* and energy *E* moving along the *x*-axis towards a finite step potential of height $U_0$ as shown in the figure. In region-I ($x < 0$), the momentum of the electron is $p_1 = \sqrt{2mE}$. The reflection coefficient at the bar-

rier is given by $R=\left(\frac{p_1-p_2}{p_1+p_2}\right)^2$, where $p_2$ is the momentum in region 2. If, in the limit $E>>U_0, R\approx\frac{v_0^2}{nE^2}$, then the integer $n$ is____.




55. A current density for a fluid flow is given by,

$$\vec{J}(x,y,z,t)=\frac{8e^t}{(1+x^2+y^2+z^2)}\hat{x}$$. At time $t=0$, the mass density $\rho(x,y,z,0)=1$.

Using the equatio of continuity $\rho(1,1,1,1)$ is found to be______ (Round off to 2 decimal places).

56. The work done in moving a $-5\mu C$ charge in an electric field $\vec{E}=(8r\sin\theta\hat{r}+4r\cos\theta\hat{\theta})V/m$, from a point $A(r,\theta)=\left(10,\frac{\pi}{6}\right)$ to a point $B(r,\theta)=\left(10,\frac{\pi}{2}\right)$, is____$mJ$.

57. A pipe of $1m$ length is closed at one end. The air column in the pipe resonates at its fundamental frequency of $400Hz$. The number of nodes in the sound wave formed in the pipe is_____.
[Speed of sound = $320m/s$]

58. The critical angle of a crystal is 30°. Its Brewster angle is_____ degrees.
(Round off to the nearest integer).

59. In an $LCR$ series circuit, a non-inductive resistor of $150\Omega$, a coil of $0.2H$ inductance and negligible resistance, and a $30\mu F$ capacitor are connected across an ac power source of $220V$, $50Hz$. The power loss across the resistor is____$W$.
(Round off to 2 decimal places).

60. A charge $q$ is uniformly distributed over the volume of a dielectric sphere of radius $a$. If the dielectric constant $\varepsilon_r=2$, then the ratio of the electrostatic energy stored inside the sphere to that stored outside is____.
(Round off to 1 decimal place)

## ANSWER KEY

| | | | |
|---|---|---|---|
| **1.** | **(b)** | **2.** | **(a)** |
| **3.** | **(c)** | **4.** | **(a)** |
| **5.** | **(b)** | **6.** | **(c)** |
| **7.** | **(b)** | **8.** | **(b)** |
| **9.** | **(d)** | **10.** | **(c)** |
| **11.** | **(c)** | **12.** | **(b)** |
| **13.** | **(c)** | **14.** | **(b)** |
| **15.** | **(d)** | **16.** | **(None)** |
| **17.** | **(b)** | **18.** | **(a)** |
| **19.** | **(d)** | **20.** | **(None)** |
| **21.** | **(a)** | **22.** | **(a)** |
| **23.** | **(c)** | **24.** | **(a)** |
| **25.** | **(d)** | **26.** | **(c)** |
| **27.** | **(d)** | **28.** | **(d)** |
| **29.** | **(b)** | **30.** | **(d)** |
| **31.** | **(a,b,c)** | **32.** | **(b,d)** |
| **33.** | **(a,d)** | **34.** | **(a,d)** |
| **35.** | **(a,b)** | **36.** | **(a,c)** |
| **37.** | **(b,c,d)** | **38.** | **(b,c,d)** |
| **39.** | **(d)** | **40.** | **(a,b,c)** |
| **41.** | **(8)** | **42.** | **(5)** |
| **43.** | **(2)** | **44.** | **(0.1)** |
| **45.** | **(6)** | **46.** | **(72)** |
| **47.** | **(0.290 – 0.31)** | **48.** | **(1.08 – 1.10)** |
| **49.** | **(2)** | **50.** | **(0.57 – 0.61)** |
| **51.** | **(0.041 – 0.045)** | **52.** | **(1.50 – 1.59)** |
| **53.** | **(537.5 – 550.2)** | **54.** | **(16)** |
| **55.** | **(2.70 – 2.74)** | **56.** | **(–1 or 1)** |
| **57.** | **(none)** | **58.** | **(27 or 63)** |
| **59.** | **(297 – 299)** | **60.** | **(0.1)** |